

**Readings** [I suggest you skim the following Wikipedia pages before attempting the problem set. These and other automatically-generated links are provided in the *Section Readings* section of the course web page]

- (1) Classical mechanics, (2) Mass, (3) Length, (4) Time in physics, (5) Pendulum, (6) Dimensional analysis, (7) Escape velocity, (8) Fermi problem, (9) Drag, (10) Taylor series, (11) Atwood machine, (12) Frustrum, (13) Trajectory, (14) Newton’s Laws of Motion

**Review** [These concepts will be useful for solving your problem set]

1. Dimensional analysis: mass = [kg], length = [m], time = [s], velocity = length / time, acceleration = velocity / time, force = mass \* acceleration
2. (Small-angle) Period of pendulum =  $2\pi \sqrt{L/g}$
3. General Taylor series:  $T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$ . where  $f^{(n)}(a)$  is the  $n$ th derivative of  $f(x)$  at  $x=a$ ;  
Taylor expansion for  $e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + \dots$
4. Approximations: most of the time you can approximate in reverse order of operation, so simplify sums and differences first (e.g.,  $a+b \approx a$  for  $a \gg b$ ) then products and ratios
5. Motion in one dimension: for most problems you only need to know and understand these 3 equations:  
 $x = x_0 + v_0 t + \frac{1}{2} a t^2$        $v^2 - v_0^2 = 2a(x - x_0)$        $v = v_0 + a t$
6. Motion in 2 dimensions: evolve in  $x$  and  $y$  directions independently. Remember:  
 $|v| = (v_x^2 + v_y^2)^{1/2}$        $v_x = v \cos(\theta)$        $v_y = v \sin(\theta)$   
 $\sin^2 \theta + \cos^2 \theta = 1$        $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
7. Newton’s second law:  $F = dp/dt$        $p = ma$   
NOTE: The formulas in #5 do not hold if  $a$  is not constant! You can use Newton’s second law and the above momentum relation to derive new ones for non-constant acceleration, if necessary.

**Exercises**

Units and dimensional analysis

1. How does the speed of waves in a fluid depend on its density,  $\rho$ , and “Bulk Modulus,”  $B$  (which has units of pressure, which is force per area)?
2. Consider a vibrating water drop, whose frequency ( $\nu$ ) depends on its radius ( $R$ ), mass density ( $\rho$ ), and surface tension ( $S$ , which is force per length). How does  $\nu$  depend on  $R$ ,  $\rho$ , and  $S$ ?

Fermi problems

3. How many notes are played on a given radio station in a given year?
4. If you stretched out all the blood vessels and capillaries in your body, how long would they be?
5. How many atoms of Caesar’s last breath do you inhale with each lungful of air?

Approximation

6. The radar used to monitor Sunday’s snowstorm relies on the relativistic Doppler effect, in which the relative motion of clouds causes a shift in the frequency of reflected radar. An observer riding in the clouds would

see frequency  $f_o = \sqrt{\frac{1 - v/c}{1 + v/c}} f_s$  for a source frequency  $f_s$ . But clouds move slowly ( $v$ ) compared to light ( $c$ )!  
What approximation can Doppler radar hardware use to calculate cloud velocity?

Motion in one dimension

7. Suppose you throw a ball vertically with speed  $V_0$  and then throw an identical ball with the same velocity after time  $T$ . When will the balls collide? Suppose the two balls collide elastically when they’re at the same height. (Assume that, since the balls are identical, the collision merely exchanges their velocities.) How long should you wait before throwing the second ball to maximize the apex height of the first?

Motion in two dimensions

8. You throw a ball with speed  $V_0$  at a vertical wall, a distance  $L$  away. At what angle should you throw the ball, so that it hits the wall at a maximum height?

Newton’s second law

9. A boat with mass  $M$  and initial speed  $V_0$  is subject to a time-dependent force  $F = A \sin(t)$  by ocean waves. How far will it travel after time  $t$ ?