

Readings

- (1) Cycloid, (2) Centripetal force, (3) Friction, (4) Atwood's machine, (5) Tension (mechanics), (6) Pulley

Review

- $(v_x, v_y) = (dx/dt, dy/dt)$  and  $(a_x, a_y) = (dv_x/dt, dv_y/dt)$
- For circular motion,  $a_{\perp} = v^2/R$
- For an object constrained to move on a circle with radius  $R$ , always  $a_{\perp} = v^2/R$  but the tangential acceleration  $a_{\parallel}$  can vary
- Static friction:  $F_{\text{static friction}} = \mu_{\text{static}} F_{\text{normal}}$  and  $F_{\text{normal}} =$  sum of forces on body perpendicular to surface.
- For a mass  $M$  on slope angle  $\theta$ , the parallel force is  $F_{\parallel} = Mg \sin \theta$  and the perpendicular force is  $F_{\perp} = Mg \cos \theta$
- Pulley problems and Atwood's machines: draw free body diagrams, write down force equations, and usually solve for tension  $T$  assuming string length is conserved. Tension on a massless string (which is the usual case) is constant.

ExercisesRelative motion

- A person is running east with a relative speed of  $v_{\text{person}}$  on a carousel with radius  $R$  and whose circumference revolves counterclockwise with speed  $v_{\text{carousel}}$ . Assume the center of the carousel lies at the origin. (a) What is the absolute velocity and speed of the runner at position  $(x, y)$ ? (b) What is the absolute acceleration of the runner at position  $(x, y)$ ? (c) Bonus (requires differential equations): If the runner starts at the westernmost point of the carousel at time  $t = 0$ , where will the runner be after time  $t$ ?

Circular motion

- What is the radius of curvature (assume constant speed along x-direction) for  $y = x^2$ ,  $y = \sin(x)$ ,  $y = e^x$ ?
- A projectile is fired at speed  $v$  and angle  $\theta$ . What is the radius of curvature of the parabolic motion (a) at the top? (b) at the beginning? (c) At what angle should the projectile be fired so that the radius of curvature at the top equals half the maximum height?
- (A lot like Problem 4!!!) A bead rests on top of a frictionless hoop of radius  $R$ , which lies in a vertical plane. The bead is given a tiny push so that it slides down and around the hoop. At what points on the hoop is the bead's acceleration purely horizontal?

Friction

- My brother's dorm held a cardboard racing competition last year (see movie). (a) Why does this process work? (b) A contestant with mass  $M$  "rides" a piece of cardboard that has static friction coefficient  $\mu_F$  with the floor and  $\mu_S$  with her sneakers by "jumping" every time  $T$  at a nearly vertical angle  $\theta$  to a raised-center-of-mass-height of  $h$ . For a given  $\theta$ , how high does the contestant need to jump in order to move forward together with the cardboard? Assume no dynamic friction.

Tension

- Let's model a stationary, heavy rope with mass  $M$  and height  $H$  hanging from the ceiling as a set of  $N$  segments with equal mass connected by joints. What is the "tension" at the joint just above the bottom  $M$  segments? How can we use this model to determine the real tension of such a rope along its height (hint: infinite number of segments)?

Pulleys

- A block with mass  $m_1$  is sliding on another block with mass  $m_2$ . The blocks are on a  $20^\circ$  slope and are connected by a massless string looped over a pulley. All surfaces are frictionless. Find the acceleration of each block and the tension in the string that connects the blocks.

