

**Readings**

(1) Normal force, (2) Newton’s laws of motion, (3) Differential equation, (4) Momentum, (5) Elastic collision

**Review**

- Remember: normal force always pushes “away” from the contact surface, and it’s not necessarily constant!
- In minimization problems, if setting the derivative to zero yields multiple solutions, some of them may in fact be maxima so check them!
- Circular motion in a slanted plane can be confusing. The trick you should use is to figure out the effective forces *in the plane*.
- Remember that if  $\dot{x} = f(x)$  then  $\int_{x_i}^{x_f} \frac{dx}{f(x)} = t_f - t_i$ .
- (Classical) momentum =  $p = \text{mass} * \text{velocity}$  and it is conserved (constant) in a closed system.
- The instantaneous thrust force on a rocket is given by  $F = v_{\text{ex}} * (dm/dt)$ , and you can assume the fuel is consumed at a constant rate. The relative velocity of rocket exhaust is relative to the rocket not the ground!
- For problems #9-10, do not break the rope up into finite elements-with-tension as we discussed in last section. Instead, treat the rope as a lumped object with the normal force exerted on the *entire rope*. In #9, assume the portion of the rope that isn’t lying flat is in free fall.

**Exercises**

Applying Newton’s Laws

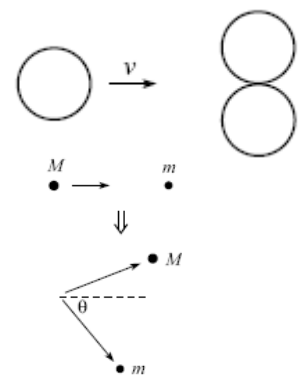
- A small block with mass  $m$  rests on a frictionless horizontal tabletop a distance  $r$  from a hole in the center of the table. A string tied to the small block passes down through the hole, and a larger block with mass  $M$  is suspended from the free end of the string. The small block is set into uniform circular motion with radius  $r$  and speed  $v$ . What must  $v$  be if the large block is to remain motionless when released?
- A wedge with angle  $\theta$  and mass  $M$  rests on a frictionless horizontal tabletop. A block with mass  $m$  is placed on the wedge and a horizontal force  $F$  is applied to the wedge. What must the magnitude of  $F$  be if the block is to remain at a constant height above the tabletop?
- Two masses start sliding at the same speed at the same positions on flat tracks that are identical except that one has a shallow dip before the finish line. Which (if any) reaches the finish line first?

First-order ordinary differential equations

- If  $\dot{v} = av^3$  and given  $v_0$ , what are  $v(t)$  and  $x(t)$ ?

Collisions

- A pool ball with initial speed  $v$  is aimed right between two other pool balls. If the two right balls leave the collision at  $30^\circ$  with respect to the initial line of motion, find the final speeds of all three balls.
- A mass  $M$  moving in the positive  $x$ -direction with speed  $v_0$  collides elastically with a stationary mass  $m$ . The collision is not necessarily head-on, so the masses may come off at angles. Let  $\theta$  be the angle of  $m$ ’s resulting motion. What should  $\theta$  be so that  $m$  has the largest possible speed in the  $y$ -direction? (Hint: Use the center of mass frame!)



More Newton’s Law

- A mass, which is free to move on a horizontal frictionless plane, is attached to one end of a massless string which wraps partially around a frictionless vertical pole of radius  $r$ . You hold onto the other end of the string. At  $t = 0$ , the mass has speed  $v_0$  in the tangential direction along the dotted circle of radius  $R$  shown. Your task is to pull on the string so that the mass keeps moving along the dotted circle. You are required to do this in such a way that the string remains in contact with the pole at all times. (You will have to move your hand around the pole, of course.) What is the speed of the mass as a function of time?

