Physics 15a – Spring 2006 – Section 6 Handout (Mar 20-21) **Alex Wissner-Gross**

Readings

(1) Center of mass, (2) Elastic collision, (3) Frame of reference, (4) Special relativity for beginners, (5) Special relativity, (6) Proper length

Review

1. Center of mass: $\mathbf{s}_{CM} = \frac{1}{m_{total}} \int \rho(\mathbf{s})\,\mathbf{s}\,dV$ where ρ(**s**) is the mass density.
2. For elastic lab frame collisions, conserve momentum and energy separately, and solve the quadratic.
3. For elastic CM frame collisions, convert to CM, negate the velocities, then convert back to lab frame.
4. Lorentz contraction: The observed length of an object with proper length L moving at speed v is $L\sqrt{1-v^2/c^2} = L/\gamma$ where $\gamma \equiv 1/\sqrt{1-v^2/c^2}$. Observed clock time also slows down by γ factor.
5. Remember the "head start" effect from lecture: in the lab frame, the "back clock" on an object with length L and velocity v will have a time delay of Lv/c^2.

Exercises

Center of mass

1. What is the center of mass of a pyramid with base side length a, height h, and uniform density?

Collisions

2. Consider the following one-dimensional collision. A mass m_1 moves to the right with speed $2v$ and a second mass m_2 moves to the right with speed v. They collide elastically. Find their final lab-frame velocities by (a) working in the lab frame, (b) working in the CM frame
3. A billiard ball collides with another one at rest. Use the following method to show that their resulting velocities are orthogonal: Start with the conservation-of-momentum equation, $m\vec{V} = m\vec{v}_1 + m\vec{v}_2$, and take the dot product of it with itself. Then combine the result with the conservation-of-energy equation, $\frac{1}{2}mV^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$, and show that the dot product of $\vec{v}_1$ and $\vec{v}_2$ is what it should be.
4. Suppose we have a stack of n balls of different masses $m_i = A^n * m_0$, as in the basketball problem (#4 on the homework). This time, the ball masses aren't necessarily so different (e.g., A=1). Find the height that the top (n^{th}) ball reaches. What happens if A=1? A<1?
5. Hold on a minute… When we change frames of reference, all the velocities change, so the total kinetic energy must change too! How can energy still be conserved?

Special relativity

6. How fast would you have to travel to see a (lab frame) stationary object next to you halve in height?
7. Train A has length L. Train B moves past A with relative speed $4c/5$, in the same direction. The length of B is such that A says that the fronts of the trains coincide at exactly the same time as the backs coincide. What is the time difference between the fronts coinciding and the backs coinciding, as measured by B?
8. Two bombs lie on a train platform, a distance L apart. As a train passes by at speed v, the bombs explode simultaneously (in the platform frame) and leave marks on the train. Due to the length contraction of the train, we know that the marks on the train will be a distance γL apart when viewed in the train's frame (since this distance is what is length-contracted down to the given distance L in the platform frame). How would someone on the train quantitatively explain to you why the marks are γL apart, considering that the bombs are only a distance L/γ apart in the train frame?
9. A train and a tunnel both have proper lengths L. The train speeds toward the tunnel, with speed v. A bomb is located at the front of the train. The bomb is designed to explode when the front of the train passes the far end of the tunnel. A deactivation sensor is located at the back of the train. When the back of the train passes the near end of the tunnel, this sensor tells the bomb to disarm itself. Does the bomb explode?