Physics 15a – Spring 2006 – Section 7 Handout (Apr 3-4) **Alex Wissner-Gross**

Readings

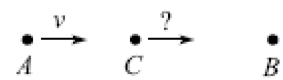
(1) Velocity-addition formula, (2) Lorentz transformation, (3) Lorentz scalar, (4) Mass in special relativity, (5) Compton scattering

Review

1. Velocity addition formula: $v_{A+B} = (v_A + v_B)/(1+v_A \cdot v_B/c^2)$
2. 1D Lorentz Transformations: $x' = \frac{x - vt}{\sqrt{1-v^2/c^2}}$, $t' = \frac{t - vx/c^2}{\sqrt{1-v^2/c^2}}$ and reverse: $x = \frac{x'+vt'}{\sqrt{1-v^2/c^2}}$, $t = \frac{t'+vx'/c^2}{\sqrt{1-v^2/c^2}}$ if the primed frame travels at *v* in lab frame.
3. Invariant (Lorentz) interval: $\tau^2 = c^2(\Delta t)^2 - (\Delta x)^2$.
4. Energy/momentum relations: $E = \gamma mc^2$, $P = \gamma mv$, $E^2 = P^2c^2 + m^2c^4$, $P/E = v/c^2$

Exercises

Velocity addition



1. *A* moves at speed *v*, and *B* is at rest, as shown. How fast must *C* travel, so that she sees *A* and *B* approaching her at the same rate? In the lab frame (*B*'s frame), what is the ratio of the distances *CB* and *AC*? The answer to this is very nice an clean. Can you think of a simple intuitive explanation for the result?

Lorentz transformation

2. A train with proper length L_1 at speed *v* enters a tunnel with length L_2. Let Event 1 be the rear of the train coinciding with the entrance of the tunnel. Let Event 2 be the front of the train coinciding with the exit of the tunnel. (a) Using length contraction, find the time separation between these two events in the ground frame. (b) Using the Lorentz transformations, find the time and space separations in the train's frame. (c) Use a length-contraction argument in the train's frame to check your answers to part (b).
3. Two deep space network nodes are launched simultaneously from Earth at different speeds $v_2 > v_1$. Right when they launch, their internal clocks are both set to zero. When Node 1 shows a time *T* it pings Probe 2 with its laser. Immediately after receiving the ping, Probe 2 pings Probe 1 back. The cycle repeats indefinitely. What are the readings of the two probe clocks each time they receive a ping?
4. Two trains of proper length L and velocities *v* and *2v* head in the same direction. How much time does it take for the trains to pass each other (defined as the time between the "fronts coinciding" event and the "backs coinciding" event): (a) In the frame of the "slower" train? (b) In the frame of the "faster" train? (c) Verify that the invariant interval between the two events is the same in both frames.

Relativistic dynamics

5. Two masses, m_1 and m_2, move in one dimension with relativistic velocities v_1 and v_2 and collide and merge to form a mass M traveling at speed V. What are M and V?
6. (Compton scattering) A photon collides with a stationary electron. If the photon scatters at an angle θ (see Figure), show that the resulting wavelength, λ', is given in terms of the original wavelength, λ, by $\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$, where *m* is the mass of the electron. Note: The energy of a photon is $E = hv = hc/\lambda$.