Readings

(1) Photon, (2) Moment of inertia, (3) Angular acceleration, (4) Angular velocity, (5) Parallel axes rule, (6) Perpendicular axes rule, (7) Greisen-Zatsepin-Kuzmin limit

Review

1. Energy/momentum Lorentz transforms: $E' = \gamma_v(E - vP)$, $P' = \gamma_v\left(P - \frac{v}{c^2}E\right)$
2. (From Lecture 15 notes) Relativistic force: $F \equiv \frac{dp}{dt} = \gamma^3 ma$
3. Angular motion: $\alpha = \frac{d\omega}{dt}$, $\omega = \frac{d\theta}{dt}$, $x_{\parallel} = R \cdot \Delta\theta$
4. Moment of inertia: $I = \int_0^M r^2 dm$
5. Parallel axis theorem: $I_{\text{parallel axis}} = I_{cm} + Md^2$ where d is distance from new axis to center of mass axis
6. Perpendicular axis theorem: for an object in x-y plane, $I_z = I_x + I_y$ if all axes pass through same point.

Exercises

Relativistic collisions

1. A photon and a mass *m* move in opposite directions. They collide head-on and create a new particle. If the total energy of the system is *E*, how should it be divided between the photon and the mass *m*, so that the mass of the resulting particle is as large as possible?
2. (Greisen-Zatsepin-Kuzmin/GZK limit) Let *E* be the energy of cosmic microwave background photons. What is the velocity *v* above which a cosmic ray with rest mass *m* will generate (assisted by the CMB) pions with rest mass π? It looks like there's a speed limit lower than *c* for long distance travel of massive particles!
3. Two masses *m* are on a collision course, each with very slow speed $v \ll c$. When they collide, they stick together. What is the first-order relativistic correction to the mass of the resulting blob?

Angular motion and moment of inertia

4. You hang a thin *square* hoop with side length *A* over a nail at the corner of the hoop. You displace it 90º to the side from its equilibrium position and let it go. What is its angular speed when it returns to its equilibrium position?
5. Find the moment of inertia of a solid cone (mass *M*, base radius *R*) around its symmetry axis.
6. Take an equilateral triangle of side *l*, and remove the "middle" triangle (1/4 of the area). Then remove the "middle" triangle from each of the remaining three triangles, and so on, forever. Let the final fractal object have mass *m*. Find the moment of inertia around a line joining a vertex to the opposite side (see right). Be careful how the mass scales.