

**Readings**

- (1) Angular momentum, (2) Rolling, (3) Torque, (4) Rotational energy

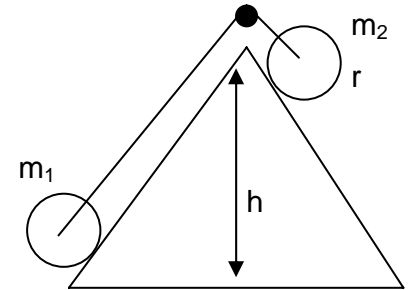
**Review**

1. Angular momentum  $L = r \times P = I \cdot \omega$
2. Non-slipping condition for object with circular cross-section:  $v = \omega \cdot R$ .
3. Rotational kinetic energy =  $\frac{1}{2} \cdot I \cdot \omega^2$
4. Torque  $\tau = r \times F = I \cdot \alpha$

**Exercises**

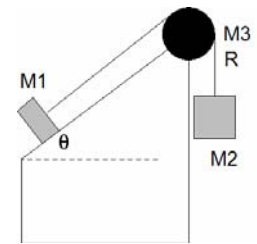
Slipping/sliding

1. Consider the system to the right, consisting of two disks with masses  $m_2 > m_1$  and radius  $r$ . Mass 2 rolls down a frictional surface without slipping while mass 1 slides up a frictionless surface. What will be the angular velocity of mass 2 after it rolls down a vertical distance  $h$ ? What is the answer if the masses are uniformly dense spheres instead of disks?



Massive pulleys

2. Consider the system to the right. The masses are  $m_1$  and  $m_2$ , and the pulley is a uniform disk of mass  $m_3$  and radius  $r$ . The string is massless and does not slip with respect to the pulley. The incline is frictionless. Find the acceleration of the masses.



Rolling on planes

3. A uniform disk with mass  $M$  and radius  $R$  sits on a piece of paper on the floor. If the paper is pulled horizontally with acceleration  $a$ , and the disk does not slip, what is the acceleration of the disk's center of mass?
- 4\*\*. A ball rolls without slipping on a perfectly flat plane of infinite extent. Will it roll forever? If not, what would stop it?

Angular collisions

5. Two disks with masses  $m_1$  and  $m_2$  and radii  $r_1$  and  $r_2$  collide inelastically with impact factor  $b$ . What is the angular velocity of the resulting blob about its center of mass?

- 6\*. Consider a hard spherical ball of radius  $a$ . Assume the bounces of this ball on a surface with friction are essentially elastic and non-slip at the point of contact. How quickly should you spin the ball when you throw it horizontally with speed  $v$  if you want it to bounce back and forth as shown to the right?

